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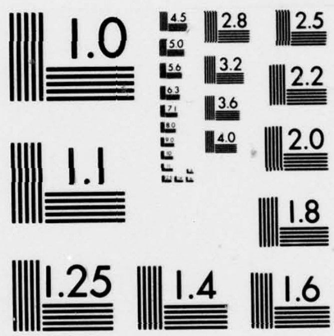
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A LINEAR MODEL FOR THE SINGLE MACHINE
LOTSIZE SCHEDULING PROBLEM

Research Report 77-2

by

Thom J. Hodgson

RESEARCH REPORT

Industrial & Systems
Engineering Department
University of Florida
Gainesville, FL. 32611

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Past research on the Bomberger lotsize scheduling problem has relied heavily on the following assumptions:

1. Each product is to be produced on a regular invariant cycle.

2. The inventory level at the onset of each production run for a product being produced is zero.

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Abstract

A Linear Model for the Single Machine
Lotsize Scheduling Problem

by

Thom J. Hodgson

Past research on the Bomberger lotsize scheduling problem has relied heavily on the following assumptions:

1. Each product is to be produced on a regular invariant cycle.
2. The inventory level at the onset of each production run for a product being produced is zero.

In this paper these two restrictions have been dropped. It is shown that the inventory cost function can still be expressed as a linear function. A linear programming model is formulated. Sensitivity analysis is used to find an optimal cycle length. Numerical examples are used to show how the model can be used to develop lotsize schedules which are both practical and low cost.

Glossary of Symbols

A_i = Set up cost of product i ($)

c_i = Cost per piece of product i ($/unit)

d_i = Demand rate of product i (pieces/unit time)

F(T) = The inventory carrying cost per unit time for a particular lotsize scheduling problem as a function of the cycle length T.

h = Inventory carrying charge ($/$inventory/unit time)

I_j = The inventory level of product k_j at the beginning of production run j.

k_j = The index of the product produced on the jth production run of the schedule

N = Number of products

n_i = The number of times product i is produced in the schedule

p_i = Production rate of product i (pieces/unit time)

s_i = Setup time of product i (time)

T = Length of the total schedule (time)

W_j = The idle time immediately after the completion of the jth production run of the cycle

X_j = The start time of the jth production run of the cycle

z_j = The index of the next production run of product k_j after production run j.

Introduction

The lotsize scheduling problem in its various forms is a problem that has plagued both practitioner and researcher. As its name implies, it has elements of both inventory and scheduling. The objective function, the sum of setup and inventory costs, is derived from inventory theory; and the feasibility requirement associated with machine availability is primarily a scheduling problem. The problem is basically that of finding a minimum cost production schedule on a single machine, for N different products, each with known demand rate, production rate, and setup time. The problem can be approached in two different ways. It can be approached as an infinite horizon problem where the objective is to determine a cyclic repeatable schedule which minimizes setup plus inventory costs per unit time. It can also be approached as a finite horizon problem where the objective is to determine a schedule over some planning period which minimizes setup plus inventory costs over that period. This paper is concerned with the former approach.

Rogers [15] provided an early comprehensive treatment of the problem. He structured the basic problem and treated the problem of schedule feasibility in great detail. Bomberger [3] developed what might be considered the first algorithmic approach (Dynamic Programming) for the problem. In addition (and possibly more important), he developed a standard data set of 10 products which has provided the yardstick by which most of the following researchers have evaluated their efforts. The literature contains a succession of papers [1, 2, 4 - 16], each providing some additional insight into the structure of the problem as defined by Bomberger.

The rather unique and insightful approach of Elmaghraby and Mallik [5] deserves comment. They structured the problem using a network model. The power of the approach lies in its conceptual simplicity and its adaptability to both finite and infinite horizon problems. An efficient computational

procedure for the approach has yet to be developed.

It should be noted that the Bomberger definition of the problem does not necessarily match what, in fact, happens on the production floor. A basic assumption is that each product is produced at a given constant interval, and that the inventory for each product reaches zero just as its production run begins. As a consequence, the sum of setup and inventory cost per unit time as a function of the cycle length T for a product is just [10]

$$\frac{A}{T} + hc(1 - d/p)d\,\frac{T}{2}, \qquad (1)$$

where A = setup cost ($),

h = inventory carrying charge ($/$ inventory/unit time),

c = piece cost ($/unit),

d = demand rate (pieces/unit time), and

p = production rate (pieces/unit time).

The least cost cycle length under these assumptions is just [10]

$$T^* = \left[\frac{2A}{hc(1 - d/p)d}\right]^{1/2} \qquad (2)$$

Equations (1) and (2) are widely known and used. However, it has been this author's experience that the basic assumptions associated with the cost structure of equation (1) are not always valid when one works on the production floor. Machine availability constraints may force the use of unequal lotsizes, staggered cycle patterns, and nonzero inventories at the beginning of production runs. This phenomenon was recognized by Maxwell [13, 14] in his early work. Baker [1] also recognized the practicality of unequal lotsizes. Maxwell formulated the lotsize scheduling problem as a mathematical program. His model requires a repeatable sequence of jobs as input with each product (i) being produced n_i times during the sequence. The formulation has linear constraints.

However, the objective is quadratic and non-convex. In general, the mathematical program is not readily solveable.

In succeeding sections of this paper, a model requiring a repeatible sequence of jobs as input is formulated in which staggered cycle patterns and non-zero inventories at the beginning of production runs are allowed. The model has linear constraints and a linear objective. Consequently, it can be optimized using linear programming. It is shown that the optimal length of the schedule (cycle time) can be found using sensitivity analysis. An example is given to show how the model can be used to find useful production plans.

Model Formulation

Consider the following situation. Product i (with demand rate d_i) is produced on a single machine n_i times every T time units. Each time it is produced, $d_i T/n_i$ units are produced. It is desired that product i be setup and produced exactly every T/n_i time units. The desired (least cost) inventory function is shown in Figure 1a. However, product i must compete for the available time on the machine with the other N - 1 products assigned to the machine. In order to achieve a feasible schedule, it is deemed necessary to move the schedule for a production run of product i ahead in time. The inventory function for this situation is shown in Figure 1b. As can be seen from Figure 1b, the inventory level only reaches the level I at the stage of the rescheduled production run and is inflated over part of the next inventory cycle. This results in an increased inventory cost for the schedule which is proportional to the shaded area in Figure 1b. Since the lotsize schedule will repeat itself every T time units, the increase in total inventory cost per unit time can be calculated by evaluating the area of the shaded area (Figure 1b), multiplying by the inventory carrying cost (h) and the piece cost (c_i), and dividing by the cycle length (T):

$$\Delta \text{ cost} = h \cdot c_i \cdot (\text{shaded area})/T.$$

The shaded area is easily seen to be equal to E · G, where

$$G = (1 - d_i/p_i)d_i T/n_i, \text{ and}$$

$$E = I/(d_i(1 - d_i/p_i)).$$

Therefore, multiplying and simplifying,

$$\Delta \text{ cost} = h \cdot c_i \cdot I/n_i \qquad (3)$$

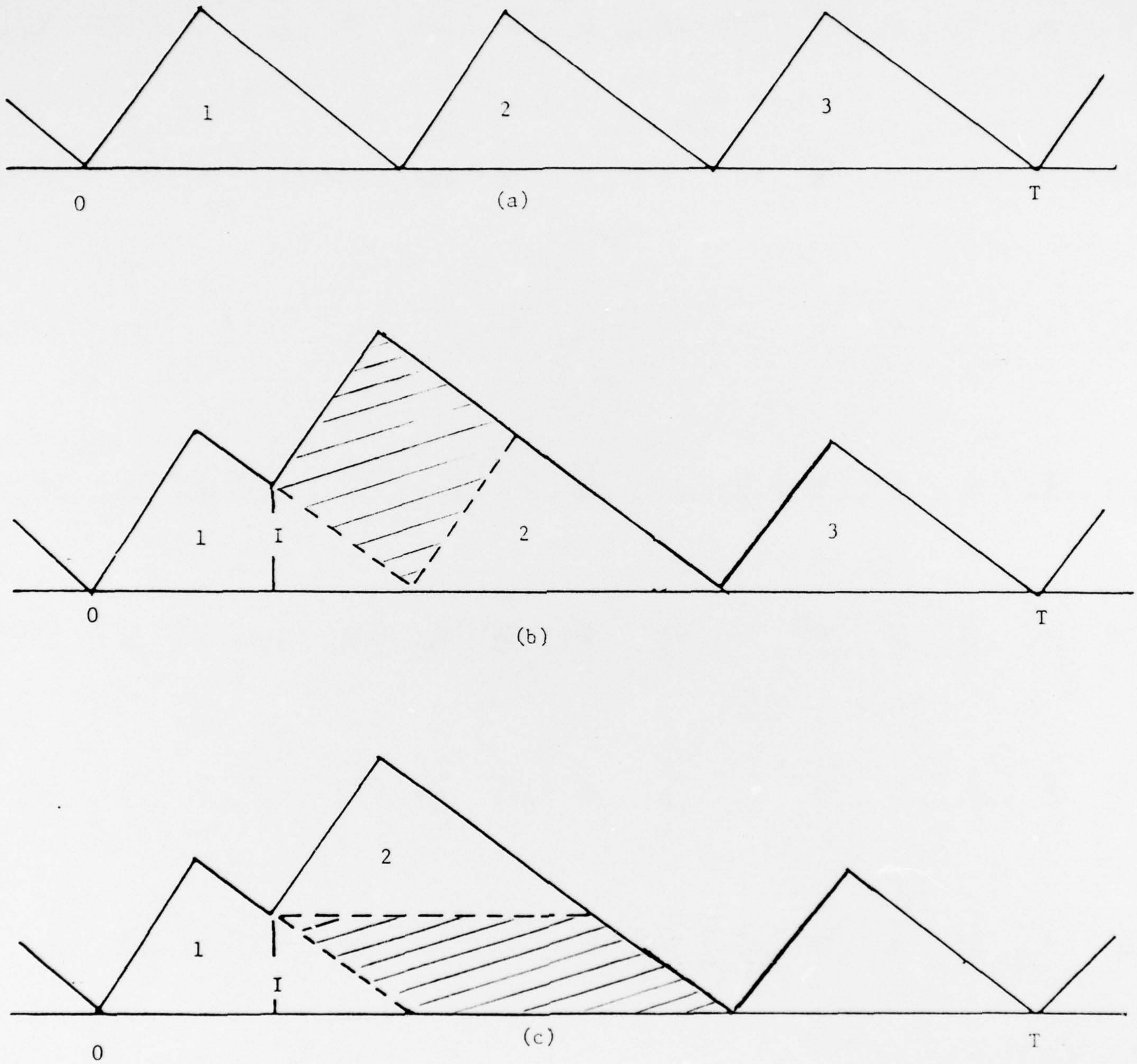

FIGURE 1

Modified Inventory Levels

Note that the increase in cost per unit time is linearly dependent on the minimum inventory level I.

With this information in hand, let us proceed to a linear formulation of the N-product, single machine, lotsize scheduling problem. Suppose that a sequence K (of length L) of production runs to be performed during a cycle (of length T time units) is given. k_j is the index of the product to be produced on the jth production run of the cycle. Let $X_j (0 < X_j < T)$ be the start time of the jth production run of the cycle, $W_j \geq 0$ be the amount of idle time immediately after the completion of the jth production run, and n_i be the number of times the index i appears in the sequence K. Note that any production run for product i lasts $d_i T/p_i n_i$ time units. The following set of equations relate the start times (X_j) and idle times (W_j).

$$X_j + \frac{d_{k_j} T}{p_{k_j} n_{k_j}} + W_j + S_{k_{j+1}} = X_{j+1} \qquad j = 1, \ldots, L - 1 \tag{4}$$

$$X_L + \frac{d_{k_L} T}{p_{k_L} n_{k_L}} + W_L = T$$

Let z_j be the index of the next production run of product k_j after production run j. Note that during any production run for product i, $d_i T/n_i$ units are produced. The following set of equations define the inventory levels (I_j) for each product at the beginning of each production run.

$$I_j + d_{k_j} T/n_{k_j} - d_{k_j}(X_{z_j} - X_j) = I_{z_j} \qquad j = 1, 2, \ldots \tag{5}$$

Note that equation (5) is not necessary when production run j is the n_{k_j}th production run for product k_j since only $n_{k_j} - 1$ equations are necessary to fully define the relationships between the n_{k_j} inventory levels.

The objective of the linear program is to minimize the total inventory cost (setup cost will be dealt with at a later point). It should be

noted that if all of the inventory levels (I_j) are zero, the only inventory costs are just the balanced cycle inventory costs.

$$h \sum_{i=1}^{N} c_i (1 - d_i/p_i) d_i T/2n_i \tag{6}$$

Any additional inventory costs due to modifying the balanced cycle schedule to achieve feasibility are accounted for by equation (3). The objective function, then, is as follows:

$$\min \{h [\sum_{i=1}^{N} c_i (1 - d_i/p_i) d_i T/2n_i + \sum_{j=1}^{L} c_{k_j} I_j / n_{k_j}]\} \tag{7}$$

Combining the objective (7) with the constraints (4) and (5), and including the usual non-negativity constraints constitutes a linear program.

Two points can be made immediately concerning the characteristics of optimal solutions to the linear program. First, it is clear at least one of the minimum inventory levels (I_j) for each product (i) will be equal to zero. This is easily proven by contradiction and is left to the reader.

Second, if T is included as a decision variable in the linear program and no bounds are placed on it, the optimal solution will occur when T is at its minimum feasible value. The minimum feasible value of T can be calculated as follows:

$$T_{min} = \frac{\sum_{i=1}^{N} n_i s_i}{1 - \sum_{i=1}^{N} d_i/p_i} \quad . \tag{8}$$

This second characteristic is difficult to prove, because as T decreases the minimum inventory levels (I_j) increase at an unspecified rate. However, it

it clear, in an intuitive sense, why T seeks a minimum value. The cost of setup is not included in the model to counter-balance the inventory cost. In the succeeding section, this problem is dealt with.

Determining the Optimal Value of T

If the feasibility constraints are ignored, the least cost value of T can be determined from the following formula:

$$T^* = \left[\frac{2 \sum_{i=1}^{N} n_i A_i}{h \sum_{i=1}^{N} c_i d_i (1 - d_i/p_i)/n_i} \right]^{1/2} . \qquad (9)$$

If the feasibility constraints are considered, it is easy to see that the optimal value of T will be no less than T^*. Therefore, as a first step to determining the optimal value of T, the following constraint can be added to the linear program:

$$T \geq \max\{T^*, T\min\} \qquad (10)$$

In order to determine the optimal value of T, it is necessary to determine the optimal value of the objective function of the linear program (inventory cost) for all values of T. As it turns out, this can be done in a straight forward fashion using sensitivity analysis. Once the inventory carrying cost function F(T) has been determined, the value of T which minimizes the sum of setup and inventory costs can be determined.

Assume that, for a particular set of products and a given sequence of production runs to be performed during the cycle, the linear program has been formulated (including constraint (10)) and solved. Assume that in order to maintain feasibility, it is necessary to schedule jobs such that several of the starting inventories are positive.

T may be increased up to a certain value and the optimal basis will still be maintained. The amount that T can be increased with no change in basis can be determined by the entries in the column of the surplus variable for constraint (10) (column v) of the final tableau (See [17], page 129-133, for a lucid explanation).

Let I_j be the basic variable in the linear program solution associated with row r of the final tableau. Then the tableau entry (a_{rv}) in the rth row of the surplus variable column represents the amount I_j will change if T changes by one unit (since the surplus variable is represented in the initial tableau as a negative unit vector, the sign of the entry (a_{rv}) is reversed). By taking the ratio of the inventory level to the tableau entry (I_j/a_{rv}) for each row, the range of T, within which the optimal basis is unchanged, can be determined. In addition, the sensitivity of the objective function to T is contained in the objective row of the surplus column. As a consequence of the above discussion, it is seen that by solving the linear program, a segment of the inventory carrying cost function can be determined (at this point it should be apparent that the inventory carrying cost function is piecewise linear), along with bounds on the range of the sement. If T is increased beyond the range of the segment, the optimal basis changes, and a new segment of the inventory carrying cost function can be established. In general, when the optimal basis changes, one or more of the basic I_j will leave the basis, and a like number of non-basic W_j will enter the basis. Since the I_j's decrease as T increases, removing I_j's from the basis and replacing them with W_j's which are not included in the objective will increase the sensitivity of the objective function from segment to segment. In other words, the inventory carrying cost function F(T) is convex.

The problem of finding the optimal value of T, then, is just the problem of minimizing the sum of the piecewise linear, convex inventory carrying cost function, F(T), and the hyperbolic setup cost function

$$\sum_{i=1}^{N} n_i A_i / T. \qquad (11)$$

The graph in Figure 2 serves to illustrate the problem. The minimum total cost will occur either at one of the breakpoints or in between breakpoints. To determine if the minimum occurs between two breakpoints, the following procedure can be used. The expression for the variable elements of the total cost function in a given segment is just

$$\text{Total Cost} = \sum_{i=1}^{N} n_i A_i / T + ST, \tag{12}$$

where S is the sensitivity to T of the linear program objective function in the segment. The minimum of (12) occurs at

$$T' = \left[\frac{\sum_{i=1}^{N} n_i A_i}{S} \right]^{1/2} \tag{13}$$

If T' falls within the boundaries of the segment, then the minimum occurs within the segment. A simple procedure can now be given for finding the optimal value for T:

1. Set the lower bound on T according to equation (10) and solve the linear program. Determine the bounds on T for the linear segment.
2. Solve for T' using equation (13). If T' is greater than the upper bound for the segment GØ TØ 3. If T' is within the bounds, STØP: T' is optimal. If T' is less than the lower bound, STØP. The lower bound is optimal.
3. Increase the lower bound on T to the upper bound of the segment and update the linear program to its new optimal solution. Determine bounds on T for the new segment. GØ TØ 2.

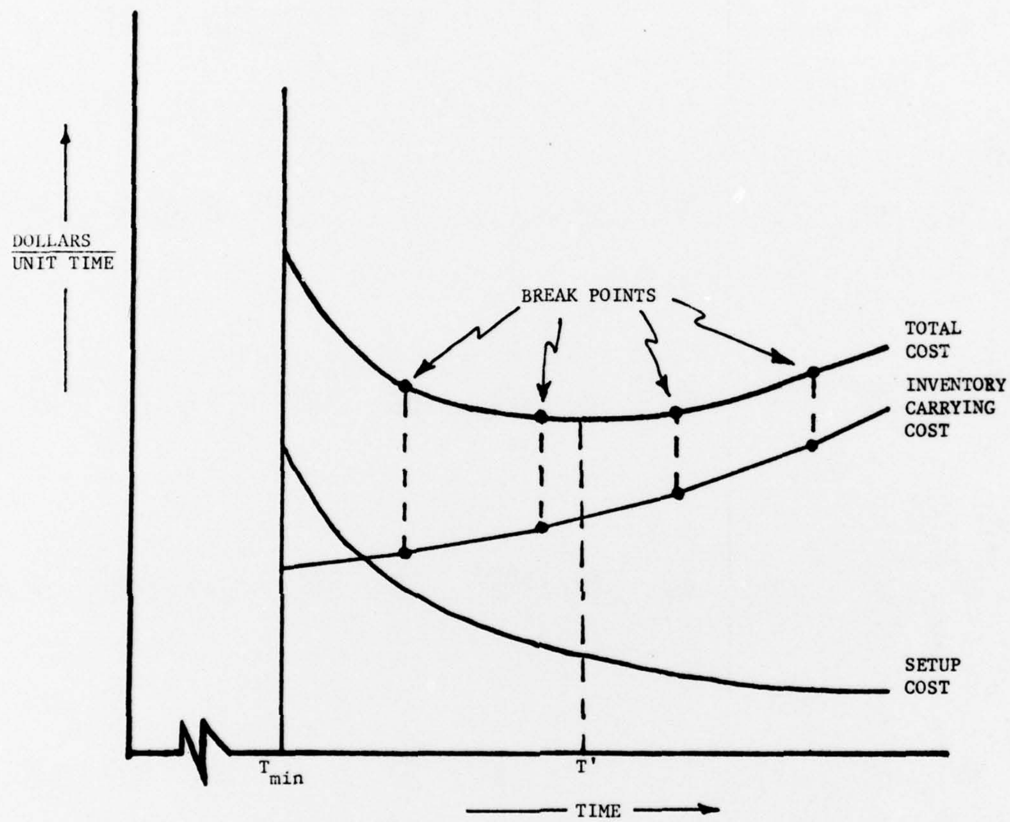

FIGURE 2

Graph of Setup and Inventory Carrying Cost Function v.s. The Cycle Length, T.

An Example

For an example let us turn to Bomberger's [3] famous ten product data set which is found in Table 1. Doll and Why bark [4] have presented the lowest cost solution to date for this data set ($32.071/day). With a total schedule length (T) of 187.395 days, products 1 through 10 are produced 1, 4, 4, 8, 4, 2, 1, 8, 4, and 4 times respectively during the cycle. A sequence of 40 jobs in Table 2, which satisfies the above cycle frequencies, was used to generate the linear program model. As expected, the procedure described in the previous section achieved the same cost as Doll and Whybark (since the sequence is feasible at T^*, no sensitivity analysis was required). However, the solution is of limited practical value. It is difficult to imagine any company that would plan to produce products every 23.424, 46.848, 93.696, or 187.395 days. It boggles one's mind to think of the reaction a production foreman might have to such a schedule.

In fact, it is not an uncommon practice [7] for companies to limit production cycles to specific lengths (i.e., one week, two weeks, one month, etc.) which can be easily implemented by production personnel. In the present case, fixing the value of T at 1 year (240 days), and fixing the individual cycle lengths of the 10 product example at 1, 3, 6, or 12 months (assuming a 20 working day month) should result in a more useable schedule. However, it is impossible to develop a feasible schedule using conventional means. By shifting production runs, the linear program is able to find a feasible solution for a cost of $32.365/day (see Table 3 for the L. P. schedule). For less than a 1% increase in setup and inventory costs, the model is able to provide a workable schedule. The point is that a production planner can use the linear programming model as a tool for the development of good practical schedules. The model does optimally what production planners and schedulers can only now do in a heuristic fashion.

Table 1

Bomberger's Data*

Part No.	Setup Time (Days)	Setup Cost ($)	Piece Cost ($/unit)	Production Rate (units/day)	Demand Rate (units/day)
1	.125	15	0.0065	30000	400
2	.125	20	0.1775**	8000	400
3	.25	30	0.1275	9500	800
4	.125	10	0.1000	7500	1600
5	.5	110	2.7850	2000	80
6	.25	50	0.2675	6000	80
7	1.0	310	1.5000	2400	24
8	.5	130	5.9000	1300	340
9	.75	200	0.9000	2000	340
10	.125	5	0.0400	15000	400

* The inventory carrying cost is .10 ($/$/year). A year consists of 240 days, 8 hours/day.

** In the original problem [3], this cost was 0.1175. All other authors used 0.1775.

Table 2
Lotsize Schedule for Doll, Whybark
Sequence

Sequence	Product	Production Start Time (Day)	Idle Time Following Production (Days)	Beginning Inventory (Pieces)
1	4	.125	0.59	0.0
2	8	6.21	0.0	0.0
3	9	13.09	0.0	0.0
4	5	21.55	0.0	0.0
5	4	23.55	0.59	0.0
6	8	29.63	0.0	0.0
7	2	35.89	0.0	0.0
8	3	38.48	0.43	0.0
9	1	42.98	0.0	0.0
10	10	45.60	0.0	0.0
11	4	46.97	0.59	0.0
12	8	53.06	0.0	0.0
13	9	59.94	0.0	0.0
14	5	68.40	0.0	0.0
15	4	70.40	0.59	0.0
16	8	76.48	0.0	0.0
17	2	82.73	0.0	0.0
18	3	85.33	0.0	0.0
19	6	89.52	1.55	0.0
20	10	92.45	0.0	0.0
21	4	93.82	0.59	0.0
22	8	99.91	0.0	0.0
23	9	106.78	0.0	0.0
24	5	115.25	0.0	0.0
25	4	117.25	0.59	0.0
26	8	123.33	0.0	0.0
27	2	129.58	0.0	0.0
28	3	132.18	0.18	0.0
29	7	137.30	0.0	0.0
30	10	139.30	0.0	0.0
31	4	140.67	0.59	0.0
32	8	146.76	0.0	0.0
33	9	153.63	0.0	0.0
34	5	162.10	0.0	0.0
35	4	164.10	0.59	0.0
36	8	170.18	0.0	0.0
37	2	176.43	0.0	0.0
38	3	179.02	0.0	0.0
39	6	183.22	1.55	0.0
40	10	186.15	0.0	0.0

Total Cost $32.071/day

Table 3
Lotsize Schedule for Practical
Lotsizes

Sequence	Product	Production Start Time (Day)	Idle Time Following Production (Days)	Beginning Inventory (Pieces)
1	4	.125	0.0	1715.90
2	8	4.89	0.0	364.63
3	9	10.87	0.0	0.0
4	4	21.20	0.0	0.0
5	8	25.96	2.13	0.0
6	2	33.45	0.0	0.0
7	5	36.95	0.0	0.0
8	10	39.47	0.0	0.0
9	4	41.20	0.0	0.0
10	8	45.96	0.0	0.0
11	3	51.44	1.65	0.0
12	6	58.40	0.0	0.0
13	4	60.12	0.0	1715.90
14	8	64.89	0.0	364.63
15	9	70.87	0.0	0.0
16	4	81.20	0.0	0.0
17	8	85.96	2.13	0.0
18	2	93.45	0.0	0.0
19	5	96.95	0.0	0.0
20	10	99.47	0.0	0.0
21	4	101.20	0.0	0.0
22	8	105.96	0.0	0.0
23	3	111.44	0.18	0.0
24	1	116.80	0.0	0.0
25	4	120.13	0.0	1715.90
26	8	124.84	0.0	364.63
27	9	130.87	0.0	0.0
28	4	141.20	0.0	0.0
29	8	145.96	2.13	0.0
30	2	153.45	0.0	0.0
31	5	156.95	0.0	0.0
32	10	159.47	0.0	0.0
33	4	161.20	0.0	0.0
34	8	165.96	0.0	0.0
35	3	171.44	1.65	0.0
36	6	178.40	0.0	0.0
37	4	180.13	0.0	1715.90
38	8	184.89	0.0	364.63
39	9	190.87	0.0	0.0
40	4	201.20	0.0	0.0
41	8	205.96	2.13	0.0
42	2	213.45	0.0	0.0
43	5	216.95	0.0	0.0
44	10	219.47	0.0	0.0
45	4	221.20	0.0	0.0
46	8	225.96	0.0	0.0
47	3	231.44	0.10	0.0
48	7	237.60	0.0	0.0

TOTAL COST $32.365/Day

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